



THE UNIVERSITY *of* EDINBURGH

Edinburgh Research Explorer

The dispersion of spherical droplets in source–sink flows and their relevance to the COVID-19 pandemic

Citation for published version:

Cummins, C, Ajayi, OJ, Mehendale, F, Gabl, R & Viola, IM 2020, 'The dispersion of spherical droplets in source–sink flows and their relevance to the COVID-19 pandemic', *Physics of Fluids*, vol. 32, no. 8, pp. 083302. <https://doi.org/10.1063/5.0021427>

Digital Object Identifier (DOI):

[10.1063/5.0021427](https://doi.org/10.1063/5.0021427)

Link:

[Link to publication record in Edinburgh Research Explorer](#)

Document Version:

Peer reviewed version

Published In:

Physics of Fluids

General rights

Copyright for the publications made accessible via the Edinburgh Research Explorer is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy

The University of Edinburgh has made every reasonable effort to ensure that Edinburgh Research Explorer content complies with UK legislation. If you believe that the public display of this file breaches copyright please contact openaccess@ed.ac.uk providing details, and we will remove access to the work immediately and investigate your claim.



Dynamics of spherical droplets in source-sink pair flow fields

C.P. Cummins,^{1,2, a)} O.J. Ajayi,¹ F.V. Mehendale,³ R. Gabl,⁴ and I.M. Viola⁴

¹⁾Maxwell Institute for Mathematical Sciences, Department of Mathematics, Heriot-Watt University, Edinburgh, EH14 4AS, UK

²⁾Institute for Infrastructure & Environment, Heriot-Watt University, Edinburgh, EH14 4AS, UK

³⁾Centre for Global Health, Usher Institute, College of Medicine and Veterinary Medicine, University of Edinburgh, EH8 9AG

⁴⁾School of Engineering, University of Edinburgh, Edinburgh, EH9 3FB, UK

(Dated: 13 July 2020)

In this paper, we investigate the dynamics of spherical droplets in the presence of a source-sink pair flow field. The dynamics of the droplets is governed by the Maxey-Riley equation with Basset-Boussinesq history term neglected. We find that, in the absence of gravity, there are two distinct behaviours for the droplets: small droplets cannot go further than a specific distance, which we determine analytically, from the source before getting pulled into the sink. Larger droplets can travel further from the source before getting pulled into the sink by virtue of their larger inertia, and their maximum travelled distance is determined analytically.

We investigate the effects of gravity, and we find that there are three distinct droplet behaviours categorised by their relative sizes: small, intermediate-sized, and large. Counterintuitively, we find that the droplets with minimum horizontal range are neither small nor large, but of intermediate size. Furthermore, we show that in conditions of regular human respiration, these intermediate-sized droplets range from a few μm to a few hundred μm . The result that such droplets have a very short range could have important implications for the interpretation of existing data on droplet dispersion.

I. INTRODUCTION

The transport of inertial particles in fluid flows occurs in many problems arising in engineering and biology, such as the build-up of microplastics in the ocean¹ and respiratory virus transmission through tract droplets²⁻⁴. The Maxey-Riley equation⁵ describes the motion of a finite-sized spherical particle in an ambient fluid flow. The equation is a representation of Newton's second law, in which the forces acting on the particle include a Stokesian drag force, an added mass force, a gravity force, the force due to the undisturbed flow, and a Basset-Boussinesq history term. The equation takes the form of a second-order, implicit integro-differential equation with a singular kernel, and with a forcing term that explodes at the starting time⁶. The equation has been applied to model the dynamics of aerosol comprising particles of various density ratios⁷, feeding mechanism of jellyfish^{8,9}, and the dynamics of inertial particles in vortical flows^{10,11}.

The Basset-Boussinesq term accounts for the drag due to the production of vorticity as the particle is accelerated from rest. It is difficult to include this term numerically, and is often omitted on the assumption that particles move in a quasistatic manner¹². This assumption breaks down in bubbly and slurry flows, where the Basset-Boussinesq term accounts for a quarter of the forces on the particle¹² when density ratio $R = 2\rho_f' / (\rho_f' + 2\rho_p')$ is greater than $2/3$, where ρ_f' is the fluid density and ρ_p' is the particle density. Recent advances¹³ have shown that the full Maxey-Riley equation can be represented as a forced, time-dependent Robin boundary condition of the 1-D diffusion equation. Here, the authors found that a particle

settling under gravity relaxes much more slowly ($t^{-1/2}$) to its terminal velocity than if the Basset-Boussinesq was neglected, where it relaxes exponentially quickly¹⁴.

In this paper, we examine the transport of inertial particles in source-sink flows¹⁵. Such a flow could represent the trajectories of water droplets emitted from coughing, sneezing²⁻⁴, or breathing and in the presence of extraction, such as an air-conditioning unit or air current. Since the dynamics of settling droplets is significantly affected by their size, it is important to understand the impact that the emitted particle size has on the destination of such a particle in a source-sink flow. In particular, since droplets are vectors for infectious diseases such as COVID-19, it is imperative that we understand the particle dynamics in such flows to mitigate the spread of the disease.

The paper is organised as follows: in Section II, the mathematical model is presented and non-dimensionalised. The results are presented in Section III, for small (III A) and intermediate-sized (III B) particles in the absence of gravity. Gravitational effects are considered for small particles in III C and for intermediate-sized particles in III D. In Section IV, we present applications for our results for human breathing without (IV B) and with (IV C) the inclusion of extraction. Finally, we discuss our findings in Section V.

II. MATHEMATICAL MODEL

Consider a source producing air of density ρ_{air}' and viscosity ν_{air}' , with volume flux of Q_1' , containing spherical liquid droplets of radius a' and density ρ_{drop}' , which are emitted with a characteristic velocity U' . Let us represent the 3D velocity field $\mathbf{u}'_{\text{source}}(\mathbf{x}')$ at a position \mathbf{x}' of the emitted air as a point

^{a)}Electronic mail: c.cummins@hw.ac.uk

source of strength Q'_1 , centred at the origin¹⁵:

$$\mathbf{u}'_{\text{source}}(\mathbf{x}') = \frac{Q'_1 \mathbf{x}'}{4\pi |\mathbf{x}'|^3}. \quad (1)$$

We include an extraction unit as a point sink of strength Q'_2 located at a position \mathbf{x}'_0 as follows:

$$\mathbf{u}'_{\text{sink}}(\mathbf{x}') = -\frac{Q'_2 (\mathbf{x}' - \mathbf{x}'_0)}{4\pi |\mathbf{x}' - \mathbf{x}'_0|^3}. \quad (2)$$

The resulting airflow is given by the linear superposition of these two flows:

$$\mathbf{u}'(\mathbf{x}') = \frac{Q'_1 \mathbf{x}'}{4\pi |\mathbf{x}'|^3} - \frac{Q'_2 (\mathbf{x}' - \mathbf{x}'_0)}{4\pi |\mathbf{x}' - \mathbf{x}'_0|^3}. \quad (3)$$

The natural timescale of the problem emerges as $T' = |\mathbf{x}'_0|/U'$. We non-dimensionalise (3) according to

$$\mathbf{x} = \mathbf{x}'/|\mathbf{x}'_0| \quad \mathbf{u} = \mathbf{u}'/U', \quad (4)$$

which gives the nondimensionalised expression for the airflow velocity

$$\mathbf{u}(\mathbf{x}) = \Lambda \left(\frac{\mathbf{x}}{|\mathbf{x}|^3} - \gamma \frac{(\mathbf{x} - \mathbf{x}_0)}{|\mathbf{x} - \mathbf{x}_0|^3} \right), \quad (5)$$

with $\Lambda = Q'_1/4\pi U'|\mathbf{x}'_0|^2$, $\gamma = Q'_2/Q'_1$, and $\mathbf{x}_0 = \mathbf{x}'_0/|\mathbf{x}'_0|$.

The velocity of the droplet embedded in this background airflow obeys the Maxey-Riley equation⁵

$$\begin{aligned} \dot{\mathbf{v}}(t) - \frac{3}{2}R \frac{D\mathbf{u}}{Dt} \Big|_{\mathbf{x}(t)} &= \left(1 - \frac{3}{2}R\right) \mathbf{g} - A(\mathbf{v}(t) - \mathbf{u}(\mathbf{X}(t), t)) \\ &\quad - \sqrt{\frac{9}{2\pi}} \frac{R}{\sqrt{St}} \left[\int_0^t \frac{\dot{\mathbf{v}}(s) - \dot{\mathbf{u}}(\mathbf{X}(s), s)}{\sqrt{t-s}} ds + \right. \\ &\quad \left. \frac{\mathbf{v}(0) - \mathbf{u}(\mathbf{X}(0), 0)}{\sqrt{t}} \right]. \end{aligned} \quad (6)$$

where $\mathbf{X}(t)$ is the position of the droplet at time t , $\mathbf{v}(t) = \dot{\mathbf{X}}(t)$ is its velocity, and

$$\begin{aligned} R &= \frac{2\rho'_{\text{air}}}{\rho'_{\text{air}} + 2\rho'_{\text{drop}}}, \quad A = \frac{R}{St}, \\ St &= \frac{2}{9} \left(\frac{a'}{|\mathbf{x}'_0|} \right)^2 Re, \quad \mathbf{g} = \frac{|\mathbf{x}'_0| \mathbf{g}'}{U'^2}, \end{aligned} \quad (7)$$

with \mathbf{g}' the acceleration due to gravity vector, $Re = U'|\mathbf{x}'_0|/\nu'_{\text{air}}$ is the Reynolds number, and St is the particle Stokes number. Note here that the Faxén correction terms⁵ have not been omitted: they are identically zero since $\Delta \mathbf{u} = \mathbf{0}$.

The approximate ratio of Basset history drag to Stokes drag is $O(St^{1/2})$, which, for the range of St we are interested in, is generally much less than one. In the remainder of the paper, we neglect the Basset history term since we anticipate its magnitude is negligible compared to the Stokes drag term for the parameters of interest to us, and the resulting equation is

$$\dot{\mathbf{v}}(t) - \frac{3}{2}R \frac{D\mathbf{u}}{Dt} \Big|_{\mathbf{x}(t)} = \left(1 - \frac{3}{2}R\right) \mathbf{g} - A(\mathbf{v}(t) - \mathbf{u}(\mathbf{X}(t), t)). \quad (8)$$

A. Computational considerations

The resulting equations (8) are a set of three coupled second-order non-linear ordinary differential equations (ODEs) for the position vector $\mathbf{X}(t)$. The algebra involved in computing the material derivative in (8) is straightforward, but cumbersome, and it is omitted here. This set of equations does not admit analytical solutions in general, so it must be solved numerically.

We solved the equations by expressing them as a system of six first-order ODEs using Matlab's ode15s, a variable-step, variable-order solver based on the numerical differentiation formulas¹⁶. This was performed on a laptop equipped with an Intel(R) Core(TM) i9-9980HK CPU (2.40GHz) and 32GB of RAM; and each trajectory took on average 0.015 seconds to compute. In each of our plots, we use 31 trajectories, giving a total simulation time of approximately 0.47 seconds.

Computational fluid dynamics (CFD) is an alternative approach to the presented mathematical model. For most applications the basic Navier-Stokes (NS) equations are simplified to Reynolds-averaged Navier-Stokes (RANS) equations by time-averaged and consequently removing the fluctuating quantities. Consequently, the computational needs can be reduced significantly in comparison to a full solution of the NS but the effects of the turbulence (Reynolds stress) has to be modelled with a specific model. The complexity of the equations still requests a numerical approximation and hence a solution based on a time and space discretisation (time step and computational grid).

To show the advantage of the presented mathematical approach, a comparable study using the commercial CFD model ANSYS-CFX was conducted. For this simplified investigation, it was assumed that the particles follow the air based on an additional transport equation. Typical calculation times for one set of parameters were between 3 to 4 hours (Processor Intel(R) Xeon(R) CPU E3-1275 v3 @ 3.50GHz and 32GB of RAM), which depends on the size of the fluid domain, applied mesh, complexity of the geometry as well as the specific additional models. Nevertheless, this comparison of required time for one run shows the significant potential of the presented approach, which allows us to identify the most important combinations out of a wide variation of variables in a computational time that is at least four orders of magnitude faster than CFD.

III. THE RESULTS

A. Small droplets in the absence of gravity

In the absence of gravity, (8) reads (dropping the explicit time dependence)

$$\dot{\mathbf{v}} - \frac{3R}{2} [\mathbf{u} \cdot \nabla \mathbf{u}] \Big|_{\mathbf{x}} = -\frac{R}{St} \left(\mathbf{v} - \mathbf{u} \Big|_{\mathbf{x}(t)} \right). \quad (9)$$

In (9), for small droplets ($St \ll R$) emitted from the source, the balance is between the first term on the left-hand side and

the right-hand side, so that the velocity rapidly adjusts to the background flow $\mathbf{v} \approx \mathbf{u}|_{\mathbf{X}(t)}$.

We are interested in whether droplets move away from or towards the sink. To this end, we look for trajectories for which $\mathbf{v} > 0$:

$$\mathbf{v} = \frac{d\mathbf{X}}{dt} > 0 \iff \frac{\mathbf{X}}{|\mathbf{X}|^3} > \gamma \frac{(\mathbf{X} - \mathbf{x}_0)}{|\mathbf{X} - \mathbf{x}_0|^3}, \quad (10)$$

If we take $\mathbf{x}_0 = [1, 0, 0]$, then the trajectory that emerges from the source and travels in the direction of the negative x -axis is the one that gets the greatest distance away from the sink. Hence, let us consider this inequality in the first component, and along the line $y = 0, z = 0$:

$$\frac{dX(t)}{dt} > 0 \iff \frac{X}{|X|^3} > \gamma \frac{(X-1)}{|X-1|^3}. \quad (11)$$

We are interested in where the flowfield changes direction, since this indicates the maximum distance droplets emitted at the source can travel before moving towards the sink. To this end, let us choose a point $x = -\lambda$ along $y = 0$ and $z = 0$; then this inequality tells us that

$$\frac{dX(t)}{dt} > 0 \iff \gamma > \left(1 + \frac{1}{\lambda}\right)^2. \quad (12)$$

This inequality can hold only if $\gamma > 1$. This makes sense, since flow is directed towards the sink only if the sink is stronger than the source.

Figure 1 shows the trajectories for small droplets ($St \ll R$) in the presence of a source-sink pair: the source is located at the origin (green disk) and the sink is located at $x = 1$ along the x -axis (red disk). For $\gamma = 1$ (Figure 1a), we have equal strength and droplets can take large excursions from the source before returning to the sink. As γ increases, the trajectories emanating from the source occupy an increasingly compacted region (Figure 1b-d). We can use this inequality above to define a region

$$|\lambda| < \frac{\sqrt{\gamma} + 1}{\gamma - 1}, \quad (13)$$

such that small droplets do not get further than a distance $|\lambda|$ before travelling towards the sink. The circle with radius $|\lambda|$ is shown in Figure 1 (dashed curve). Observe that, as one gets increasingly close to the source ($\lambda \rightarrow 0$), the inequality tends to

$$\frac{dX(t)}{dt} > 0 \iff \gamma > \frac{1}{\lambda^2}, \quad (14)$$

meaning that, in order to maintain trajectories moving away from a given test point, the sink strength needs to increase quadratically with distance of the test point to the source.

B. Intermediate-sized droplets in the absence of gravity

For $St = O(R)$ and $St \gg R$, the particle is slowed down exponentially according to

$$\mathbf{v}(t) \approx \mathbf{v}(0) \exp[-(R/St)t], \quad (15)$$

which represents a balance between inertia and drag forces. Provided $\gamma > 1$, and in the absence of gravity, in the long-term, the particle will always migrate towards the sink. However, in the case of intermediate-sized droplets, the maximum distance travelled by the droplet before it moves towards the sink is given by $|\mathbf{v}(0)|/(R/St)$. Since the initial velocity of the droplet is chosen to be the same as the surrounding fluid, then we can write the maximum distance as $|\mathbf{u}(\mathbf{X}(0), 0)|/(R/St)$. In our non-dimensionalisation, our characteristic velocity U' was chosen to be that of the outlet. Hence, in this non-dimensionalisation, $|\mathbf{u}(\mathbf{X}(0), 0)| = 1$. Since we also set our initial condition on the velocity to be the same as the fluid's velocity at the source, then we have the following condition on $\mathbf{X}(0)$:

$$|\mathbf{X}(0)| = \sqrt{\Lambda}. \quad (16)$$

Figure 2 shows the trajectories of intermediate-sized droplets for $\gamma = 5$ in the absence of gravity. The striking feature of the plot is the shift from a regime where the maximal extent of the trajectories as predicted by (13) is no longer valid and must be replaced with a circle of radius St/R .

C. The effect of gravity on small droplets

As droplets move from the source to the sink, gravity attempts to pull them vertically downwards. Over the timescale of the problem: i.e., the average time it takes a droplet to travel from source to sink, gravity may or may not have an appreciable effect. Intuitively, one would imagine that smaller droplets are influenced more by the airflow than gravity: for stronger sinks, the effect of gravity is comparatively less. Also, intuitively, one would think that this holds true provided that the source and sink are not too far away.

For $St \ll R, R < 2/3$, and in the absence of gravity, there are three fixed points: the source, sink and a saddle point located at $x = -|\lambda|$ along the x -axis (Figure 1). When gravitational effects are included, the fixed point at $x = -|\lambda|$ moves clockwise along about the origin as the effect of gravity is increased (see Figure 3a). A fourth fixed point (saddle) is created far from the source-sink pair, which gradually moves towards the sink (Figure 3b,c) as the effect of gravity is increased. In Figure 3d, the separatrices (indicated as blue dashed curves) show that there is a wedge of trajectories that escape the pull of the sink. As might be expected, these trajectories are those that point directly away from the sink.

D. The effect of gravity on intermediate-sized droplets

Small droplets are deflected by gravity, but generally feel the pull of the sink. Whether or not they are pulled in is determined by the interaction of gravity, the angle of their trajectory, and the strength of the sink. As the droplets become larger, gravitational effects dominate, and the sink becomes ineffective. In Figure 4, we show how the droplet trajectories

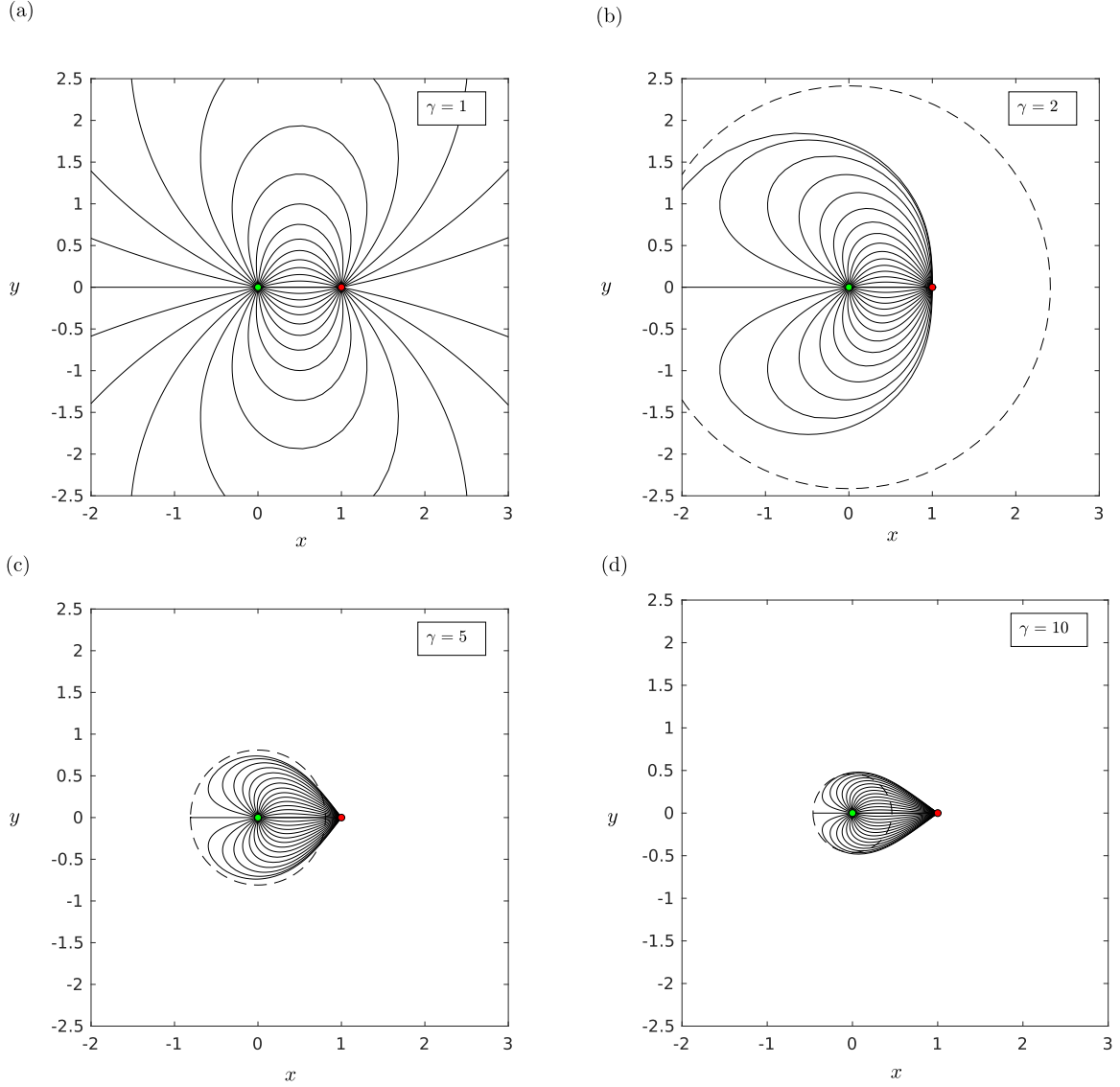


FIG. 1. The trajectories $\mathbf{X}(t)$ in the xy plane of small droplets $St \ll R$ with a background source-sink pair of various strengths γ . In these plots, $R = 0.001$, $\Lambda = 0.0001$, $|\mathbf{g}| = 0$. The trajectories do not change for changing R . The dashed circle indicates the predicted maximal distance that a particle can travel in this regime, calculated using the inequality (13).

behave as St is increased. Figure 4a, show the familiar situation where the droplets are so small that gravity does not appreciably affect their trajectory.

As gravity is increased, Figure 4b shows that there are a range of trajectories with ejection angles α (defined with respect to the positive sense of the x -axis) around the source that are deflected downwards away from the sink. This is consistent with previous sections. However, at a critical St , each ejection angle is deflected downwards by gravity (Figure 4c). In this case, the maximum horizontal distance travelled by the droplets is very small. Interestingly, this trend is not monotonic. Further increasing St , the trajectories adopt a ballistic trajectory (Figure 4d). Such droplets can move in very close proximity to the sink, but are not pulled into it (Figure 4d).

IV. EXAMPLES OF APPLICATION

A. Background on Respiratory Virus Transmission

One of the possible applications of this paper, is to underpin more sophisticated analytical or numerical models to study the transmission of respiratory viruses. In medical applications, it is common practice to categorise the emitted fluid particles as larger droplets from $5 \mu\text{m}$ to 1 mm in diameter that have a ballistic trajectory, and aerosol that remains airborne. Droplets smaller than $5 \mu\text{m}$ and desiccated droplet nuclei are known as aerosol, which can remain airborne for several hours^{17–19}. Respiratory viruses are transmitted from virus-laden fluid particles to the recipient through (1) aerosol inhalation; or (2) droplet deposition on the recipient's mouth, nose or conjunc-

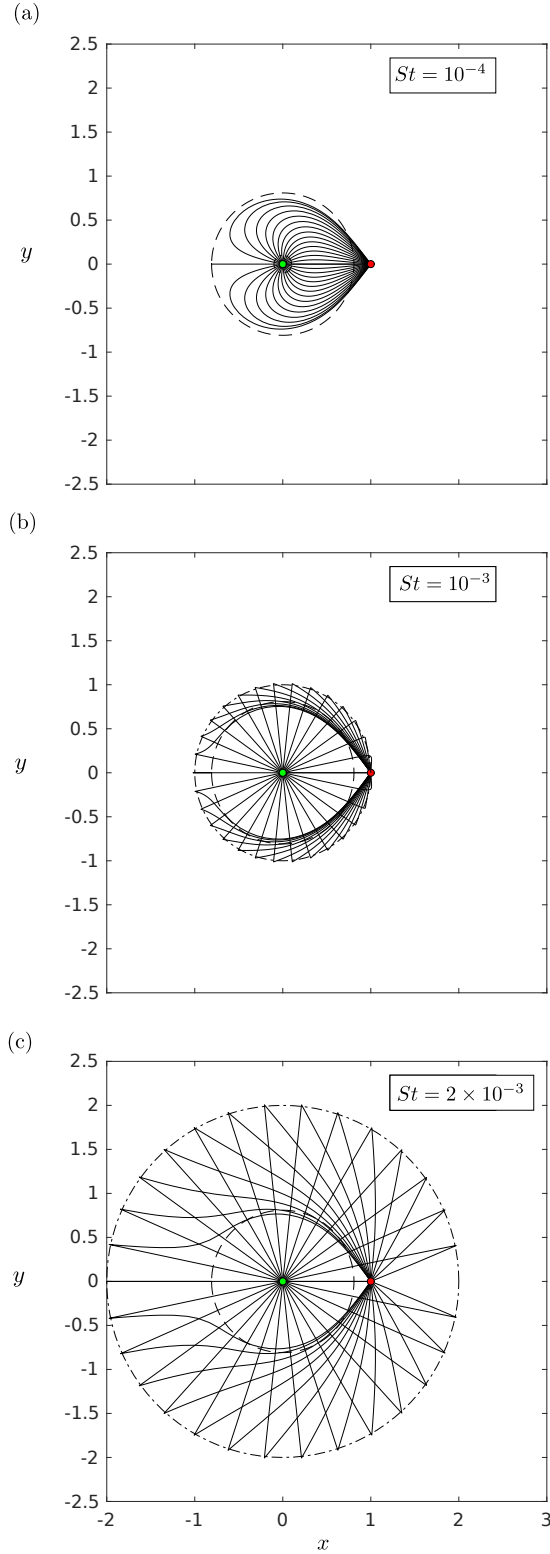


FIG. 2. The trajectories $\mathbf{X}(t)$ in the xy plane of droplets with a background source-sink pair with strength ratio $\gamma = 5$ for various values of St . In these plots, $R = 0.001$, $\Lambda = 0.0001$, $|\mathbf{g}| = 0$. The dashed circle indicates the predicted maximal distance that a particle can travel in this regime, calculated using the inequality (13). The dash-dotted circle indicates the maximal distance predicted by inertia-drag balance, giving radius equal to St/R .

tiva; or (3) droplet deposition on a surface and successive transmission through physical contact²⁰. The SARS-CoV-2 virus, for example, has a diameter of 70–90 nm²¹ and it is carried by droplets and aerosol^{19,22}.

The model proposed in this paper can provide new insights on the aerosol transmission, i.e. through those particles whose Stokes number is not sufficiently large to have a ballistic trajectory. The relative importance of aerosol (1) and droplet (2 and 3) virus transmission is not always known, and it is yet to be established for the SARS-CoV-2²³. Counterintuitively, it has been argued that aerosol could be more dangerous than larger droplets²⁴. Smaller droplets ($\leq 5 \mu\text{m}$) suspended in aerosol might carry a higher concentration of virus than larger droplets ($> 5 \mu\text{m}$)^{23,25,26}. The largest droplets are less likely to penetrate deeply in the respiratory system and might be deactivated by the effective first structural and defence barrier of the mucosa²⁷. Conversely, aerosolised virus half-life exceeds one hour¹⁹ and can be transported airborne through inhalation deep into the lungs^{28–31}, avoiding the defences of the upper respiratory system. Furthermore, aerosol inoculation has been shown to cause more severe symptoms than droplets administered by intranasal inoculation and the dose of influenza required for inoculation by the aerosol route is 2-3 orders of magnitude lower than the dose required by intranasal inoculation^{2,26,31}.

To apply our model to aerosol dispersion, we consider the particles ejected by a person talking. A person ejects about tens of fluid particles per second with diameters between³² $0.1 \mu\text{m}$ to 1 mm and with a speed of the order³³ of 1 m s^{-1} . Because this is the most frequent source of aerosol, this accounts for most of the aerosol inhaled by other people^{34,35}. Coughing leads to the ejection of 100-1000 fluid particles per second with a speed around 10 m s^{-1} , while sneezing generates 1000-10,000 fluid particles per second with a speed of up to³⁶ 20 m s^{-1} . The values presented in this paragraph should be taken as indicative because there is a significant variability between different experimental studies^{2,26,29,31,37–48}.

Some of the physics that is not considered in this work, is the particle-particle interaction and evaporation. In fact, fluid particles are ejected through a jet that transports particles in the range of $2 \mu\text{m} - 150 \mu\text{m}$ ^{38,49,50}, i.e. the aerosol, while the largest droplets have a ballistic trajectory independent of the surrounding flow^{2,50,51}. The jet can be either laminar or turbulent when breathing and speaking, while coughing and sneezing always results in a turbulent jet with a diameter-based Reynolds number higher² than 10^4 . Once ejected, the air jet extends along a straight trajectory; its diameter increases linearly with the travelled distance, while the mean velocity linearly decreases, and the turbulent statistics remain constant (i.e. the jet is self similar⁵²). Once the largest particles with a ballistic trajectory have left the air jet, the jet bends upwards due to the buoyancy force caused by the temperature and thus density difference². Smaller size particles ($\leq 100 \mu\text{m}$) are transported by the jet while they evaporate. Once a droplet exits the jet, it falls at its settling speed. For a particle with a diameter of $50 \mu\text{m}$ and $10 \mu\text{m}$, the settling speed is less than 0.06 m s^{-1} and 0.03 m s^{-1} , respectively. The smallest of these two droplets is likely to land in the form of a desiccated

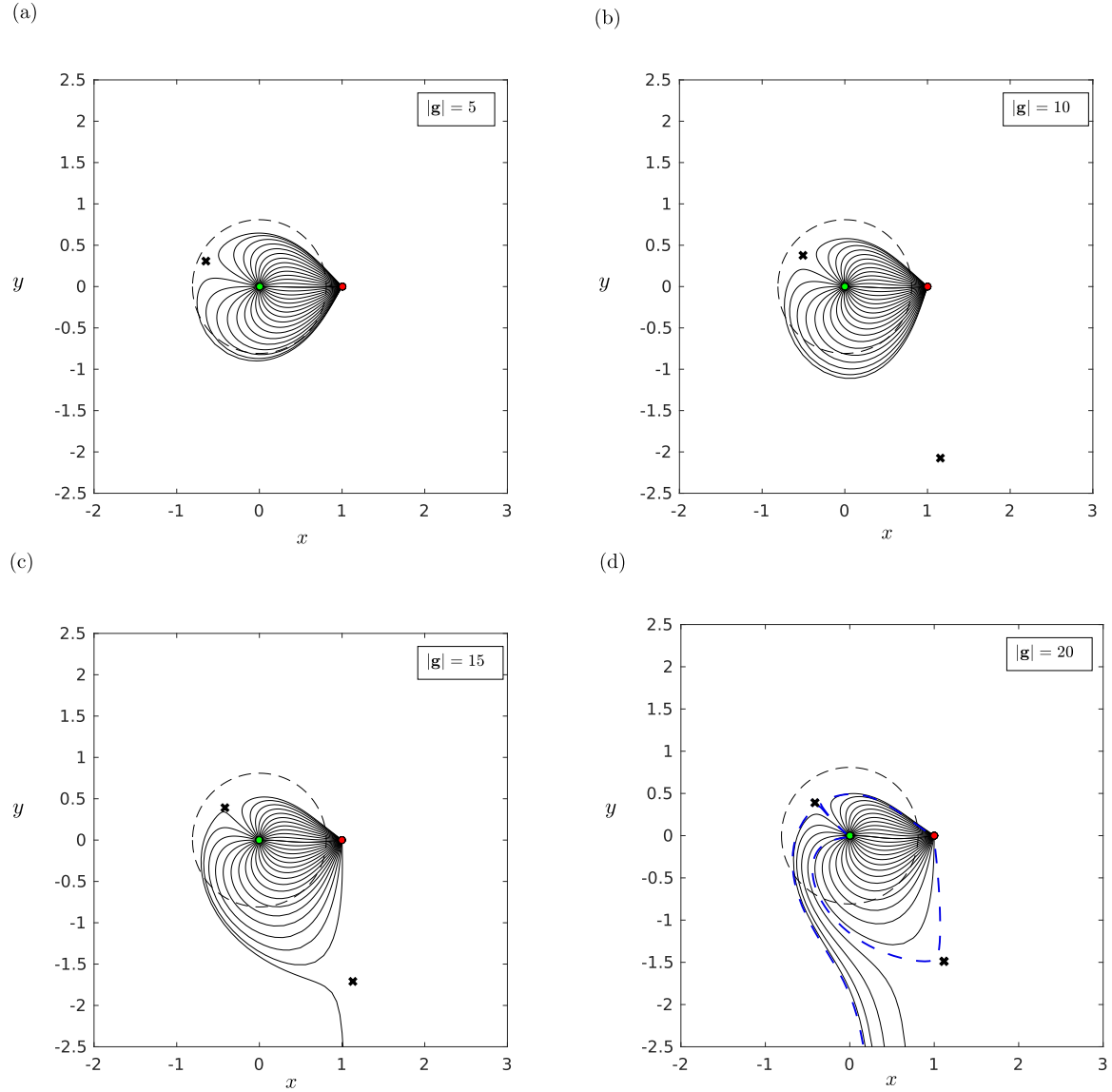


FIG. 3. The trajectories $\mathbf{X}(t)$ in the xy plane of small droplets $St \ll R$ with a background source-sink pair with strength ratio $\gamma = 5$ and for various strengths of gravity. In these plots, $R = 0.001$, $\Lambda = 0.0001$. The trajectories will be different for different choices of R . The dashed circle indicates the predicted maximal distance that a fluid parcel can travel when ejected from the source. The black crosses indicate the position of saddle fixed points.

nucleus. In fact, while a droplet with a diameter of $50 \mu\text{m}$ evaporates in about 6 s, a $10 \mu\text{m}$ droplet evaporates in less than^{2,53} 0.1 s. Once these droplets leave the jet, they can still be transported by ambient air currents, which have speeds typically in excess⁵⁴ of 0.01 m s^{-1} . These currents are modelled by the sink-source flow field discussed in this paper.

A key issue that is discussed in this study is the extent to which the cloud of droplets and aerosol are displaced into the neighbouring environment, as this is associated with virus transmission risk. Previous studies estimated that the overall horizontal range of the droplets generated while breathing and coughing before they land on the ground is around 1-2 m^{49–51}. These studies led to the CDC⁵⁵ and WHO⁵⁶ social distancing guidelines. Nonetheless, the complex physics in-

involved, which includes knowledge of the particle size distribution, their speed of evaporation, the viral charge of droplets of different size, the diffusivity of the virus-laden particles, etc., makes it difficult to assess which is the effective dispersion of virus-laden fluid particles into the environment once ejected. It was found that the largest droplets generated by sneezing can reach a distance as high as 8 m ^{2,3,50}, while aerosol dispersion is highly dependent on the temperature, humidity and air currents. For these reasons, this paper does not aim to provide definitive measures for the aerosol displacements but contributes to building a body of evidence around this complex question.

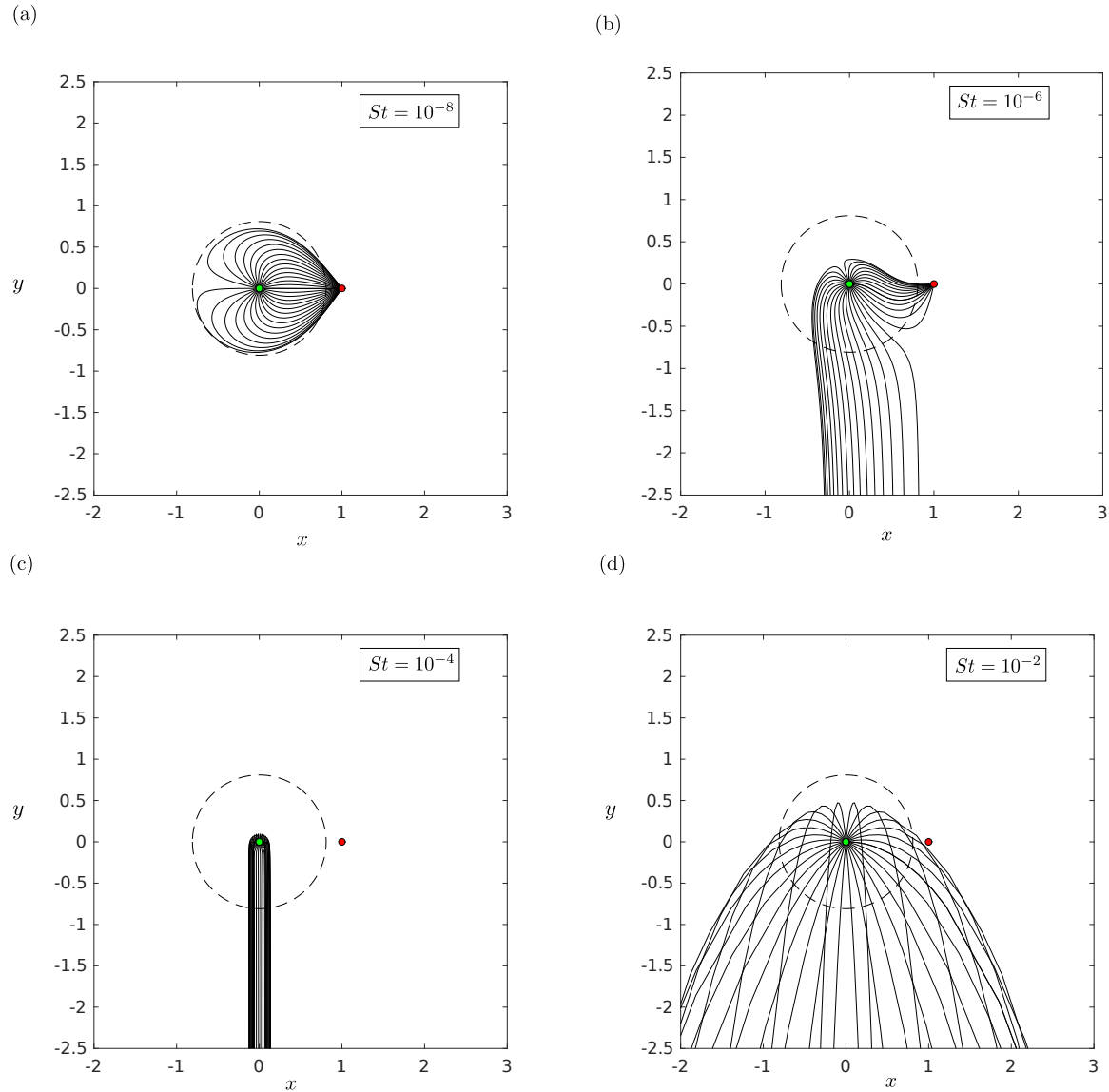


FIG. 4. The trajectories $\mathbf{X}(t)$ in the xy plane with a background source-sink pair with strength ratio $\gamma = 5$ and for various Stokes numbers St . In these plots, $R = 0.001$, $\Lambda = 0.0001$, and $|\mathbf{g}| = 1$. The trajectories will be different for different choices of R . The dashed circle indicates the predicted maximal distance that a fluid parcel can travel when ejected from the source.

B. Predicted Droplet Dispersion

Currently, there is a large amount of disagreement in the reported spectra of droplet sizes in respiratory events². The analysis is complicated by various factors including the evaporation of the droplets as they travel from the source, which in turn, is influenced by ambient humidity and temperature. Recent mathematical modelling of droplet emission during talking have categorised droplets into one of three groups⁵⁷: small ($< 75\mu\text{m}$), intermediate ($75\text{--}400\mu\text{m}$) and large ($> 400\mu\text{m}$). Small droplets approximately follow the air and can travel a great distance by weakly feeling the effects of gravity. Large droplets can also travel a large distance due to their inertia. However, the intermediate-sized particles feel strongly both gravity and drag, and their trajectory is a complex interaction

of these effects. Similar trends were observed in computational fluid dynamics simulations of previous authors⁵⁸.

In this section, we examine the problem from a much simplified perspective: we ignore evaporation entirely. We model the situation as a point source emitting droplets of various sizes in the presence of gravitational forces, and compute the maximum horizontal distance travelled by these droplets. In this case, $Q'_2 = 0 \text{ L min}^{-1}$, and other quantities such as jet speed, direction and spread are taken from recent experimental studies of the authors⁵⁹: these quantities are summarised in Table I.

We find that for both heavy and quiet breathing, the maximum distance travelled by droplets depends strongly on the droplet diameter – see Figure 5. As expected, small droplets can travel many metres, however, we see that there is an

Quantity	Description	Value	Units
U'	Jet velocity (quiet) [†]	0.55	m s ⁻¹
	Jet velocity (heavy) [†]	4.97	m s ⁻¹
α	Jet angle (direction) [†]	-5.8	°
β	Jet angle (spread) [†]	29.2	°
ρ'_{air}	Density of air	1.149	kg m ⁻³
ρ'_{drop}	Density of droplet	1000	kg m ⁻³
ν'_{air}	Viscosity of air	16.36×10^{-6}	m ² s ⁻¹
Q'_1	Volume influx (quiet) [†]	23.8	L min ⁻¹
	Volume influx (heavy) [†]	133	L min ⁻¹
Q'_2	Volume outflux	0	L min ⁻¹
$ \mathbf{x}'_0 $	Characteristic length [‡]	0.5	m

TABLE I. Physical quantities for dispersion of droplets. [†] parameters taken from previous experimental study⁵⁹ and [‡] taken from wind tunnel experiments⁴⁸.

intermediate range of droplet diameters where the horizontal distance is minimised. For quiet breathing, this minimum occurs between $69\mu\text{m} < d < 76\mu\text{m}$, while for heavy breathing this minimum occurs between $50\mu\text{m} < d < 56\mu\text{m}$. This multi-modal behaviour is reminiscent of that in previous experimental studies that measured the size distributions of droplets in various respiratory events such as talking and coughing^{47,48} and sneezing³⁶. The multi-modal behaviour observed in experiments is attributed to the different generation modes: bronchiolar, laryngeal and oral. In our simplified model, we do not have any assumption on the biological origin of the droplet: the existence of the minimum is a characteristic of the *droplets themselves* and cannot be used as an indicator of the underlying particle size distribution.

In order to unpick the physics, observe that the drag force scales as the diameter of the droplet, but the weight of the droplet scales as the diameter cubed, hence for large droplets, the drag force is negligible in comparison with the inertia of the particle. As shown before, the droplets are slowed down exponentially in the horizontal direction and are accelerated in the vertical direction by gravity, giving the maximum horizontal range of the droplet (when nominally $Y = -1$)

$$X = (St/R) \left(1 - \exp \left[-(R/St) \sqrt{\frac{2}{(1 - \frac{3}{2}R)|g|}} \right] \right). \quad (17)$$

For large particles ($St \gg R$) we can then estimate that the maximum distance L is

$$L \approx \sqrt{\frac{2}{(1 - \frac{3}{2}R)|g|}}, \quad (18)$$

meaning that the trajectories are ballistic, and we expect that for $St \gg R$, the maximum distance becomes independent of St , in agreement with the observation that large droplets' trajectories are independent of the surrounding flow^{2,50,51}.

For small particles $St \ll R$, the drag decreases linearly with droplet diameter, but the weight rapidly decreases cubically with decreasing diameter. Hence small particles follow the airflow faithfully with little influence from gravity. Such droplets can get great distances before falling, as shown in the left-hand side of Figure 5.

In the case of small droplets, the horizontal component of the droplet's trajectory follows the airflow like a tracer, and the droplet falls at its Stokesian settling velocity. Upon inspection, we find that the maximum horizontal distance L (when nominally $Y = -1$) tends to the following asymptote as $St \rightarrow 0$.

$$L = \left(\frac{2A\Lambda}{(1 - \frac{3}{2}R)|g|} \right)^{1/3}. \quad (19)$$

We can therefore estimate that droplets for which $L > 1$, or equivalently

$$St < \frac{2R\Lambda}{(1 - \frac{3}{2}R)|g|}. \quad (20)$$

(i.e., the droplets travel farther in the horizontal direction than the vertical direction) weakly feel gravity.

In between these two extreme cases, the drag force on the droplet is of the same magnitude as the gravitational force. By balancing these two effects, we can approximate the upper bound of St where the droplets become ballistic:

$$St < \frac{R}{(1 - \frac{3}{2}R)|g|}. \quad (21)$$

Such droplets are not light enough to get carried any great distance by the ambient airstream, but do not have large enough inertia to become ballistic.

Hence, we have the following designations:

(I) small droplets with St satisfying $St < \frac{2R\Lambda}{(1 - \frac{3}{2}R)|g|}$, which act like fluid tracers

(II) intermediate-sized droplets with $\frac{2R\Lambda}{(1 - \frac{3}{2}R)|g|} < St < \frac{R}{(1 - \frac{3}{2}R)|g|}$.

(III) large droplets with $St > \frac{R}{(1 - \frac{3}{2}R)|g|}$, which adopt ballistic trajectories.

This is illustrated in Figure 6, where the black curves are the numerical solutions to quiet (a) and heavy (b) breathing at zero direction and spread angle⁵⁹ and the red dashed curves indicate the expressions in (18) for large St and (19) for small St . The black vertical lines indicate the distinction between small and intermediate-sized (see (20)) and intermediate-sized and large droplets (see (21)).

Reverting to dimensional quantities, we have the following range of intermediate-sized droplets.

$$\sqrt{\frac{9\nu'_{\text{air}}Q'_1\rho'_{\text{air}}}{\pi g'|\mathbf{x}'_0|^2(\rho'_{\text{air}} - \rho'_{\text{drop}})}} < d' < 2\sqrt{\frac{9\nu'_{\text{air}}\rho'_{\text{air}}U'}{2g'(\rho'_{\text{drop}} - \rho'_{\text{air}})}}. \quad (22)$$

Plugging in the numbers from Table I, we have the approximate range:

$$3\mu\text{m} < d' < 138\mu\text{m}, \quad (23)$$

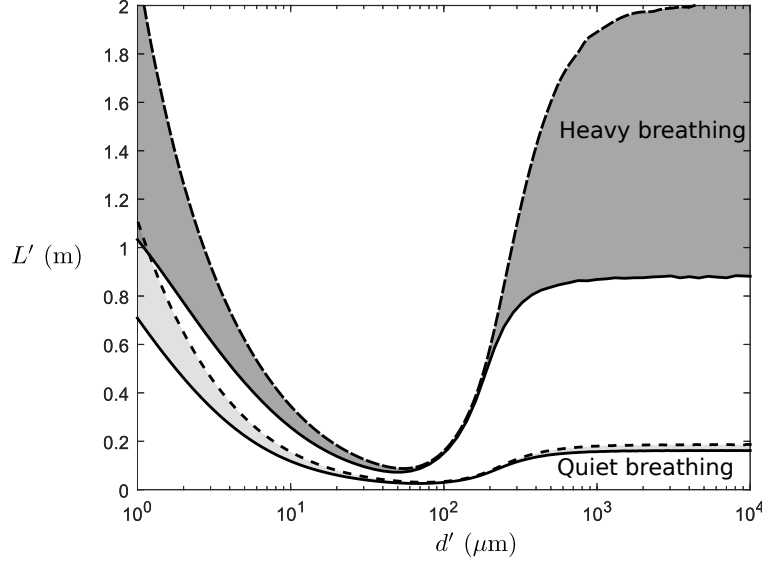


FIG. 5. The maximum distance (L') travelled for droplets of various diameters (d') with quiet (light grey) and heavy (dark grey) breathing. The dashed curve corresponds to trajectories with ejection angle equal to $\alpha + \beta/2$, while the solid curve corresponds to trajectories with ejection angle equal to $\alpha - \beta/2$ in Table I.

for quiet breathing and

$$7\mu\text{m} < d' < 414\mu\text{m}, \quad (24)$$

for heavy breathing. Our upper bound is in good agreement with previous categorisations of droplets⁵⁷, although our lower bound seems to be smaller than those found by previous authors.

Quantity	Description	Value	Units
U'	Breath jet velocity	1	m s^{-1}
ρ'_{air}	Density of air	1.149	kg m^{-3}
ρ'_{drop}	Density of droplet	1000	kg m^{-3}
ν'_{air}	Viscosity of air	16.36×10^{-6}	$\text{m}^2 \text{s}^{-1}$
Q'_1	Volume influx	6.5	L min^{-1}
Q'_2	Volume outflux	2600	L min^{-1}
$ \mathbf{x}'_0 $	source-sink distance	0.2	m

TABLE II. Physical quantities for extraction.

C. The effectiveness of extraction on droplets

Consider a person breathing air of density $\rho'_{\text{air}} = 1.149 \text{ kg m}^{-3}$ and kinematic viscosity $\nu'_{\text{air}} = 16.36 \times 10^{-6} \text{ m}^2 \text{s}^{-1}$ containing water droplets of density $\rho'_{\text{drop}} = 1000 \text{ kg m}^{-3}$. In human respiration^{32,38}, the exhaled droplets have diameters $2a'$ in the range $0.5 \mu\text{m}$ to $2000 \mu\text{m}$. For a human breathing at rest, their average volume flux is in the range $Q'_1 = 5\text{--}8 \text{ L min}^{-1}$: these values of flow rate are similar to those in previous studies⁶⁰, which reports 13 litre/min for breathing, and the typical speed of a jet in normal breathing conditions is of the order of $U' = 1 \text{ m s}^{-1}$. In violent respiratory events, such as sneezing or coughing, these values could be significantly higher². Finally, the extraction unit is located a distance of $|\mathbf{x}'_0| = 0.2 \text{ m}$ from the person. These quantities are summarised in Table II.

Based on these numbers, the non-dimensional parameters that govern the trajectory of the particle are determined to be $R = 0.00115$, $Re = 12,225$, $1.70 \times 10^{-12} < St < 0.017$, $0.067 < A < 6.76 \times 10^8$, $\Lambda = 0.00068$, and $|\mathbf{g}| = 1.96$ where the ranges indicate the values attained for the stated droplet size distribution. The parameter γ relates the flux of the extraction unit to the flux of a human's breath, and its effect will

be examined. In particular, if we suppose that the envisaged extraction unit has a volume flux approximately equal to that of a standard vacuum cleaner (100 cfm), then we can approximate that $\gamma \approx 400$. In Figure 7, we show the efficacy of such extraction for a range of St . Extraction is very effective at low $St < 10^{-4}$, however for $St > 4.2 \times 10^{-4}$, such extraction is ineffective. This upper bound of the Stokes number corresponds to water droplets of diameter 0.16 mm . Droplets larger than this will not be collected by extraction. In the nomenclature of Section IV B, the effective range of extraction corresponds to non-ballistic droplets.

V. DISCUSSION AND CONCLUSION

In this paper, we have presented a simplified mathematical model for droplet dispersion from a source and in the presence an aerosol extractor. In the absence of gravity, and for $St \ll R$, droplets behave as ideal tracers and the maximum distance that they can travel before being extracted is a function of γ only. In this case, there are two (source, and sink if $\gamma = 1$)

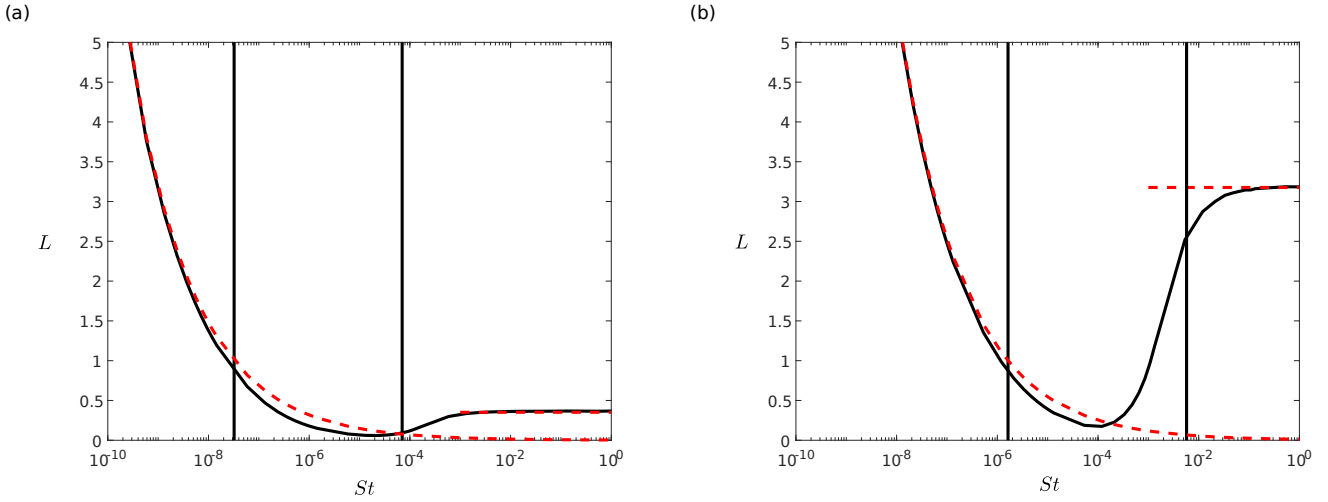


FIG. 6. The maximum distance (L) travelled for droplets of various St with quiet breathing (a) and heavy breathing (b). The vertical solid lines indicate the distinction between small and intermediate-sized (from (19)) and from intermediate-sized to large (from (21)).

or three (source, sink, and saddle if $\gamma > 1$) fixed points. The fixed points in this study are co-linear, and the position of the saddle depends on γ alone, for any given distance between source and sink. For moderate St , the droplets' inertia carry them far away from the source until they are slowed down by drag forces and pulled into the sink. In this case, the maximum distance that droplets can travel is given by R/St .

When gravity effects are taken into account, the saddle point for $St \ll R$ is no longer co-linear but moves on an arc, clockwise about the source, and a fourth fixed point (saddle) emerges approximately below the sink fixed point. For fixed γ , this fixed point moves closer to the source as the magnitude of gravity is increased. In this case, there is a set of trajectories that are pulled away from the sink by gravity. For moderate St , gravity plays an increasingly important role, and there is a critical value of gravity that pulls all trajectories vertically downwards away from the source. For yet larger St , the trajectories adopt a ballistic trajectory, with even those that travel close to the sink not being pulled in.

We included simulations relevant to human respiration, as well as simulations to inform the development of an aerosol extractor for use in clinical settings. These models can help to guide recommendations on maximum safe distances between source and sink. Additionally, these models provide a better understanding of the behaviour of individual droplets of various sizes, that may be present in a wide range of aerosols contaminated with viruses or other pathogens. This may help clinicians to make better informed decisions regarding safety while performing aerosol generating procedures; and in their choices of the type of PPE they wear. Lastly, these models provide a basis upon which aerosol and droplet contamination from a wide range of surgical, medical, dental and veterinary AGPs can be modelled, while taking into account airflows in confined clinical spaces. In this case, we found that for $St \leq 10^{-4}$, all of the aerosol is extracted and that gravity has minimal effect, this St corresponds to droplets with approximate diameter equal to 0.08 mm. Droplets larger than this

are affected by gravity, and for $St = 10^{-2}$, corresponding to droplets equal to 0.78 mm, none of the droplets are extracted. Such large droplets would be typically captured by of personal protective equipment (PPE), such as FFP1 masks, which have pore sizes typically smaller than $1\mu\text{m}$, corresponding to a droplet Stokes number of about 10^{-7} .

We determined the maximum range of droplets ejected from the source in the absence of a sink, and found that the range is minimised for intermediate-sized droplets. We find that in human respiration, this pertains to droplets within the observed range of ejected droplets. This could have implications for the interpretation for data coming from experiments on biological subjects. In particular, those that attribute observed bi- and tri-modal droplet dispersion to biological functions. Our studies suggest that the bi-modal nature of the curve is a function of the droplet's Stokes number and not necessarily linked to a specific biological function.

In our model, we neglected the Basset history term and the added mass term in the Maxey-Riley equation. These Basset history term is of significant importance for bubbly flows, where it can account for a quarter of the instantaneous force on a bubble¹². Generally speaking, for $R \ll 2/3$, this term can be safely ignored for small and intermediate-sized droplets. Recent studies have also shown that neglecting it in modelling of raindrop growth leads to a substantial overestimate of the growth rate of the droplet. Hence, for the solutions that become ballistic, we expect that such trajectories would be influenced by the Basset history term, and should be included. To do this efficiently, there is a very promising method developed recently¹³. Since this is not the focus of our study (such droplets can be captured by other forms of PPE), we do not perform such a study here.

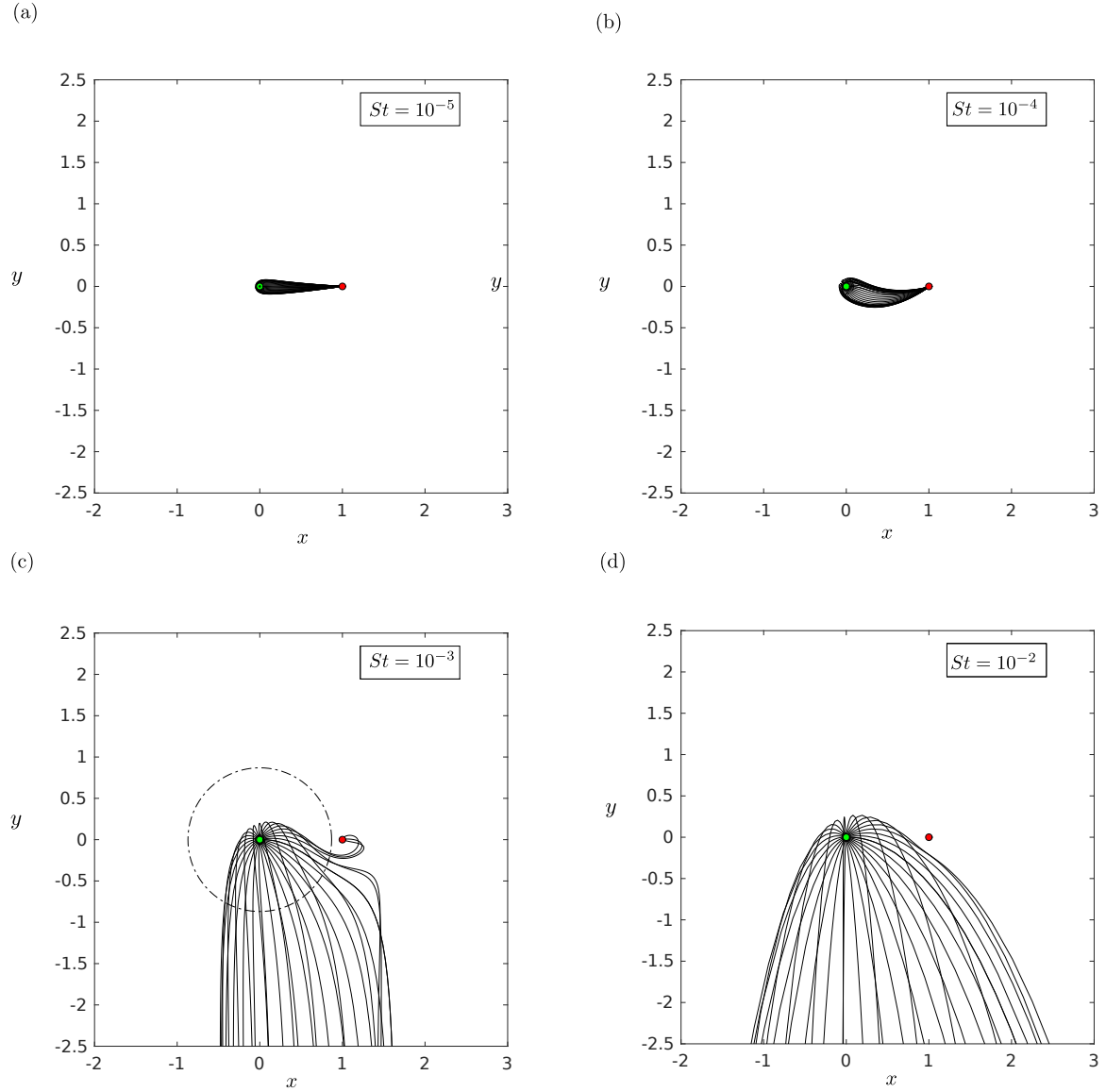


FIG. 7. The trajectories $\mathbf{X}(t)$ in the xy plane with a background source-sink pair with strength ratio $\gamma = 400$ and for various Stokes numbers St . In these plots, $R = 0.00115$, $\Lambda = 0.00068$, and $|\mathbf{g}| = 1.96$. The dashed circle indicates the predicted maximal distance that a fluid parcel can travel when ejected from the source.

ACKNOWLEDGEMENTS

OJA acknowledges the support of the Commonwealth Scholarship Commission in supporting this research.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

¹F. J. Beron-Vera, M. J. Olascoaga, and P. Miron, “Building a Maxey-Riley framework for surface ocean inertial particle dynamics,” *Physics of Fluids* **31**, 096602 (2019), arXiv:1906.01080.

²L. Bourouiba, E. Dehandschoewercker, and J. W. Bush, “Violent expiratory events: On coughing and sneezing,” *Journal of Fluid Mechanics* **745**, 537–563 (2014).

³L. Bourouiba, “Turbulent Gas Clouds and Respiratory Pathogen Emissions: Potential Implications for Reducing Transmission of COVID-19,” *JAMA - Journal of the American Medical Association*, E1–E2 (2020).

⁴S. Verma, M. Dhanak, and J. Frankenfield, “Visualizing the effectiveness of face masks in obstructing respiratory jets,” *Phys. Fluids - Invit. (in Revis.)* **061708**, 1–7 (2020).

⁵M. R. Maxey and J. J. Riley, “Equation of motion for a small rigid sphere in a nonuniform flow,” *Physics of Fluids* **26**, 883–889 (1983).

⁶M. Farazmand and G. Haller, “The Maxey-Riley equation: Existence, uniqueness and regularity of solutions,” *Nonlinear Analysis: Real World Applications* **22**, 98–106 (2015).

⁷M. R. Maxey, “The motion of small spherical particles in a cellular flow field,” *Physics of Fluids* **30** (1987), 10.1063/1.866206.

⁸J. H. Costello and S. P. Colin, “Morphology, fluid motion and predation by the scyphomedusa *Aurelia aurita*,” *Marine Biology* **121**, 327–334 (1994).

- ⁹J. Peng and J. O. Dabiri, “Transport of inertial particles by Lagrangian coherent structures: Application to predator-prey interaction in jellyfish feeding,” *Journal of Fluid Mechanics* **623**, 75–84 (2009).
- ¹⁰O. A. Druzhinin, “Concentration waves and flow modification in a particle-laden circular vortex,” *Physics of Fluids* **6**, 3276–3284 (1994).
- ¹¹N. Raju and E. Meiburg, “Dynamics of small, spherical particles in vortical and stagnation point flow fields,” *Physics of Fluids* **9** (1997).
- ¹²E. E. Michaelides, “Review—the transient equation of motion for particles, bubbles, and droplets,” *Journal of Fluids Engineering, Transactions of the ASME* **119**, 233–247 (1997).
- ¹³S. G. Prasad, V. Vasan, and R. Govindarajan, “Accurate solution method for the Maxey-Riley equation, and the effects of Basset history,” *Journal of Fluid Mechanics* **868**, 428–460 (2019).
- ¹⁴G. P. Langlois, M. Farazmand, and G. Haller, “Asymptotic Dynamics of Inertial Particles with Memory,” *Journal of Nonlinear Science* **25**, 1225–1255 (2015), arXiv:1409.0634.
- ¹⁵G. K. Batchelor, *An Introduction to Fluid Dynamics* (Cambridge University Press, 2000).
- ¹⁶L. F. Shampine, M. W. Reichelt, and J. A. Kierzenka, “Solving index-I DAEs in MATLAB and Simulink,” *SIAM Review* **18** (1999).
- ¹⁷M. Nicas, W. W. Nazaroff, and A. Hubbard, “Toward understanding the risk of secondary airborne infection: Emission of respirable pathogens,” *Journal of Occupational and Environmental Hygiene* **2**, 143–154 (2005).
- ¹⁸R. Tellier, “Review of Aerosol Transmission of Influenza A Virus,” *Emerging Infectious Diseases* **12**, 1657–1662 (2006).
- ¹⁹N. van Doremalen, T. Bushmaker, D. H. Morris, M. G. Holbrook, A. Gamble, B. N. Williamson, A. Tamin, J. L. Harcourt, N. J. Thornburg, S. I. Gerber, J. O. Lloyd-Smith, E. de Wit, and V. J. Munster, “Aerosol and Surface Stability of SARS-CoV-2 as Compared with SARS-CoV-1,” *New England Journal of Medicine*, NEJMc2004973 (2020).
- ²⁰R. M. Jones and L. M. Brosseau, “Aerosol Transmission of Infectious Disease,” *Journal of Occupational and Environmental Medicine* **57**, 501–508 (2015).
- ²¹J. M. Kim, Y. S. Chung, H. J. Jo, N. J. Lee, M. S. Kim, S. H. Woo, S. Park, J. W. Kim, H. M. Kim, and M. G. Han, “Identification of coronavirus isolated from a patient in Korea with covid-19,” *Osong Public Health and Research Perspectives*, 3–7 (2020).
- ²²Y. Liu, Z. Ning, Y. Chen, M. Guo, Y. Liu, N. K. Gali, L. Sun, Y. Duan, J. Cai, D. Westerdahl, X. Liu, K. Xu, K.-f. Ho, H. Kan, Q. Fu, and K. Lan, “Aerodynamic analysis of SARS-CoV-2 in two Wuhan hospitals,” *Nature* (2020), 10.1038/s41586-020-2271-3.
- ²³N. H. L. Leung, D. K. W. Chu, E. Y. C. Shiu, K.-H. Chan, J. J. McDevitt, B. J. P. Hau, H.-L. Yen, Y. Li, D. K. M. Ip, J. S. M. Peiris, W.-H. Seto, G. M. Leung, D. K. Milton, and B. J. Cowling, “Respiratory virus shedding in exhaled breath and efficacy of face masks,” *Nature Medicine* **2020**, 1–5 (2020).
- ²⁴R. Zhang, Y. Li, A. L. Zhang, Y. Wang, and M. J. Molina, “Identifying airborne transmission as the dominant route for the spread of COVID-19,” *Proceedings of the National Academy of Sciences*, 202009637 (2020).
- ²⁵D. K. Milton, M. P. Fabian, B. J. Cowling, M. L. Grantham, and J. J. McDevitt, “Influenza Virus Aerosols in Human Exhaled Breath: Particle Size, Culturability, and Effect of Surgical Masks,” *PLoS Pathogens* **9** (2013), 10.1371/journal.ppat.1003205.
- ²⁶W. G. Lindsley, J. D. Noti, F. M. Blachere, R. E. Thewlis, S. B. Martin, S. Othumpangat, B. Noorbakhsh, W. T. Goldsmith, A. Vishnu, J. E. Palmer, K. E. Clark, and D. H. Beezhold, “Viable influenza virus in airborne particles from human coughs,” *Journal of Occupational and Environmental Hygiene* **12**, 107–113 (2015).
- ²⁷W. Fokkens and R. Scheeren, “Upper airway defence mechanisms,” *Paediatric Respiratory Reviews* **1**, 336–341 (2000).
- ²⁸J. Atkinson and World Health Organization, *Natural ventilation for infection control in health-care settings* (World Health Organization, 2009) p. 106.
- ²⁹G. Zayas, M. C. Chiang, E. Wong, F. MacDonald, C. F. Lange, A. Senthilselvan, and M. King, “Cough aerosol in healthy participants: fundamental knowledge to optimize droplet-spread infectious respiratory disease management,” *BMC Pulmonary Medicine* **12**, 11 (2012).
- ³⁰J. Yan, M. Grantham, J. Pantelic, P. J. B. De Mesquita, B. Albert, F. Liu, S. Ehrman, and D. K. Milton, “Infectious virus in exhaled breath of symptomatic seasonal influenza cases from a college community,” *Proceedings of the National Academy of Sciences of the United States of America* **115**, 1081–1086 (2018).
- ³¹W. G. Lindsley, F. M. Blachere, R. E. Thewlis, A. Vishnu, K. A. Davis, G. Cao, J. E. Palmer, K. E. Clark, M. A. Fisher, R. Khakoo, and D. H. Beezhold, “Measurements of airborne influenza virus in aerosol particles from human coughs,” *PLoS ONE* **5** (2010), 10.1371/journal.pone.0015100.
- ³²S. Asadi, A. S. Wexler, C. D. Cappa, S. Barreda, N. M. Bouvier, and W. D. Ristenpart, “Aerosol emission and superemission during human speech increase with voice loudness,” *Scientific Reports* **9**, 2348 (2019).
- ³³J. W. Tang, A. D. Nicolle, C. A. Klettner, J. Pantelic, L. Wang, A. B. Suhaimi, A. Y. L. Tan, G. W. X. Ong, R. Su, C. Sekhar, D. D. W. Cheong, and K. W. Tham, “Airflow Dynamics of Human Jets: Sneezing and Breathing - Potential Sources of Infectious Aerosols,” *PLoS ONE* **8**, e59970 (2013).
- ³⁴J. Fiegel, R. Clarke, and D. A. Edwards, “Airborne infectious disease and the suppression of pulmonary bioaerosols,” *Drug Discovery Today* **11**, 51–57 (2006).
- ³⁵M. P. Atkinson and L. M. Wein, “Quantifying the Routes of Transmission for Pandemic Influenza,” *Bulletin of Mathematical Biology* **70**, 820–867 (2008).
- ³⁶Z. Y. Han, W. G. Weng, and Q. Y. Huang, “Characterizations of particle size distribution of the droplets exhaled by sneeze,” *Journal of the Royal Society Interface* **10** (2013), 10.1098/rsif.2013.0560.
- ³⁷J. P. Duguid, “The numbers and the sites of origin of the droplets expelled during expiratory activities,” *Edinburgh medical journal* **52**, 385–401 (1945).
- ³⁸J. P. Duguid, “The size and the duration of air-carriage of respiratory droplets and droplet-nuclei,” *Epidemiology and Infection* **44**, 471–479 (1946).
- ³⁹R. G. Loudon and R. M. Roberts, “Relation between the Airborne Diameters of Respiratory Droplets and the Diameter of the Stains left after Recovery,” *Nature* **213**, 95–96 (1967).
- ⁴⁰R. S. Papineni and F. S. Rosenthal, “The Size Distribution of Droplets in the Exhaled Breath of Healthy Human Subjects,” *Journal of Aerosol Medicine* **10**, 105–116 (1997).
- ⁴¹L. Morawska, “Droplet fate in indoor environments, or can we prevent the spread of infection?” *Indoor Air* **16**, 335–347 (2006).
- ⁴²L. Morawska, G. Johnson, Z. Ristovski, M. Hargreaves, K. Mengersen, S. Corbett, C. Chao, Y. Li, and D. Katoshevski, “Size distribution and sites of origin of droplets expelled from the human respiratory tract during expiratory activities,” *Journal of Aerosol Science* **40**, 256–269 (2009).
- ⁴³S. Yang, G. W. Lee, C.-M. Chen, C.-C. Wu, and K.-P. Yu, “The Size and Concentration of Droplets Generated by Coughing in Human Subjects,” *Journal of Aerosol Medicine* **20**, 484–494 (2007).
- ⁴⁴C. Chao, M. Wan, L. Morawska, G. Johnson, Z. Ristovski, M. Hargreaves, K. Mengersen, S. Corbett, Y. Li, X. Xie, and D. Katoshevski, “Characterization of expiration air jets and droplet size distributions immediately at the mouth opening,” *Journal of Aerosol Science* **40**, 122–133 (2009).
- ⁴⁵B. E. Scharfman, A. H. Techet, J. W. M. Bush, and L. Bourouiba, “Visualization of sneeze ejecta: steps of fluid fragmentation leading to respiratory droplets,” *Experiments in Fluids* **57**, 24 (2016).
- ⁴⁶V. Stadnytskyi, C. E. Bax, A. Bax, and P. Anfinrud, “The airborne lifetime of small speech droplets and their potential importance in SARS-CoV-2 transmission,” *Proceedings of the National Academy of Sciences*, 202006874 (2020).
- ⁴⁷X. Xie, Y. Li, H. Sun, and L. Liu, “Exhaled droplets due to talking and coughing,” *Journal of the Royal Society Interface* **6** (2009), 10.1098/rsif.2009.0388.focus.
- ⁴⁸G. R. Johnson, L. Morawska, Z. D. Ristovski, M. Hargreaves, K. Mengersen, C. Y. Chao, M. P. Wan, Y. Li, X. Xie, D. Katoshevski, and S. Corbett, “Modality of human expired aerosol size distributions,” *Journal of Aerosol Science* **42**, 839–851 (2011).
- ⁴⁹W. F. Wells, “On Air-Borne Infection Study II: Droplets And Droplet Nuclei,” *American Journal of Epidemiology* **20**, 611–618 (1934).
- ⁵⁰X. Xie, Y. Li, A. T. Y. Chwang, P. L. Ho, and W. H. Seto, “How far droplets can move in indoor environments ? revisiting the Wells evaporation/falling curve,” *Indoor Air* **17**, 211–225 (2007).
- ⁵¹J. Wei and Y. Li, “Enhanced spread of expiratory droplets by turbulence in a cough jet,” *Building and Environment* **93**, 86–96 (2015).

- ⁵²B. R. Morton, G. I. Taylor, and J. S. Turner, “Turbulent gravitational convection from maintained and instantaneous sources,” *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* **234**, 1–23 (1956).
- ⁵³H. Holterman, *Kinetics and evaporation of water drops in air*, IMAG rapport No. 2003-12 (IMAG, 2003).
- ⁵⁴A. Melikov and J. Kaczmarczyk, “Air movement and perceived air quality,” *Building and Environment* **47**, 400–409 (2012).
- ⁵⁵CDC, “Social Distancing,” (2020).
- ⁵⁶WHO, “Clinical management of severe acute respiratory infection (SARI) when COVID-19 disease is suspected,” Tech. Rep. March (World Health Organisation, 2020).
- ⁵⁷W. Chen, N. Zhang, J. Wei, H. L. Yen, and Y. Li, “Short-range airborne route dominates exposure of respiratory infection during close contact,” *Building and Environment* **176** (2020), 10.1016/j.buildenv.2020.106859.
- ⁵⁸S. W. Zhu, S. Kato, and J. H. Yang, “Study on transport characteristics of saliva droplets produced by coughing in a calm indoor environment,” *Building and Environment* **41**, 1691–1702 (2006).
- ⁵⁹I. Viola, B. Peterson, G. Pisetta, G. Pavar, H. Akhtar, F. Menoloascina, E. Mangano, K. Dunn, R. Gabl, A. Nila, *et al.*, “Face coverings, aerosol dispersion and mitigation of virus transmission risk,” *arXiv preprint arXiv:2005.10720* (2020).
- ⁶⁰J. K. Gupta, C.-H. Lin, and Q. Chen, “Characterizing exhaled airflow from breathing and talking,” *Indoor Air* **20**, 31–39 (2010).